

# Class XI Session 2025-26

## Subject - Mathematics

### Sample Question Paper - 1

Time Allowed: 3 hours

Maximum Marks: 80

#### General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

#### Section A

1.  $\sin 36^\circ = ?$  [1]  
a)  $\frac{\sqrt{10+2\sqrt{5}}}{4}$  b)  $\frac{(\sqrt{5}-1)}{4}$   
c)  $\frac{\sqrt{10-2\sqrt{5}}}{4}$  d)  $\frac{(2\sqrt{5}-1)}{4}$
2. Domain of  $\sqrt{a^2 - x^2}$  ( $a > 0$ ) is [1]  
a)  $(-a, 0]$  b)  $(-a, a)$   
c)  $[0, a]$  d)  $[-a, a]$
3. A box contains 6 nails and 10 nuts. Half of the nails and half of the nuts are rusted. If one item is chosen at random, the probability that it is rusted or is a nail is [1]  
a)  $\frac{5}{16}$  b)  $\frac{11}{16}$   
c)  $\frac{14}{16}$  d)  $\frac{3}{16}$
4.  $\lim_{x \rightarrow \infty} \frac{|x|}{x}$  is equal to [1]  
a) 0 b) -1  
c) 2 d) 1
5. The equations of the lines through  $(-1, -1)$  and making angles of  $45^\circ$  with the line  $x + y = 0$  are [1]





- a)  $\sqrt{3}$  b)  $-\sqrt{2}$   
 c)  $-\sqrt{3}$  d)  $\sqrt{2}$
17. If  $z_1$  and  $z_2$  are non-real complex numbers such that  $|z_1| = |z_2|$  and  $\text{Amp. } z_1 + \text{Amp. } z_2 = \pi$ , then  $z_1 =$  [1]  
 a)  $z_2$  b)  $-\overline{z_2}$   
 c)  $\overline{z_2}$  d)  $-z_2$
18. In an examination there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answer correct is [1]  
 a) 63 b) 11  
 c) 27 d) 12
19. **Assertion (A):** The coefficient of  $x^r$  in  $(1+x)^n$  is  ${}^nC_r$ . [1]  
**Reason (R):** The number of dissimilar terms in the expansion of  $(1-3x+3x^2-x^3)^{20}$  is 61.  
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.  
 c) A is true but R is false. d) A is false but R is true.
20. **Assertion (A):** The mean deviation about the mean for the data 4, 7, 8, 9, 10, 12, 13, 17 is 3. [1]  
**Reason (R):** The mean deviation about the mean for the data 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 is 8.5.  
 a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.  
 c) A is true but R is false. d) A is false but R is true.

### Section B

21. Is this relation function? Give reason. In case of a function, find its domain and range:  $g = \{(1,1), (1, -1), (4,2), (9,3), (16,4)\}$ . [2]

OR

If  $A = \{1, 2, 4\}$  and  $B = \{1, 2, 3\}$ , represent the set graphically:  $B \times B$ .

22. Find the derivative from the first principle:  $3x^2 + 2x - 5$ . [2]  
 23. Find the centre and radius of the circle  $2x^2 + 2y^2 - x = 0$ . [2]

OR

Find the equation of hyperbola having Foci  $(0, \pm 13)$  and the conjugate axis is of length 24.

24. If  $A = \{a, b, c, d, e\}$ ,  $B = \{a, c, e, g\}$  and  $C = \{b, e, f, g\}$ , verify that:  $A - (B \cap C) = (A - B) - (A - C)$  [2]  
 25. Find the equation of the line passing through the point  $(0, 3)$  and perpendicular to the line  $x - 2y + 5 = 0$ . [2]

### Section C

26. Find the domain and range of  $f(x) = |2x - 3| - 3$  [3]  
 27. Solve the inequality  $3x - 7 > 5x - 1$  for real  $x$ . [3]  
 28. Verify that  $(0, 7, -10)$ ,  $(1, 6, -6)$  and  $(4, 9, -6)$  are the vertices of an isosceles triangle. [3]

OR

The vertices of the triangle are  $A(5, 4, 6)$ ,  $B(1, -1, 3)$  and  $C(4, 3, 2)$ . The internal bisector of angle  $A$  meets  $BC$  at  $D$ . Find the coordinates of  $D$  and the length  $AD$ .

29. Find the middle term in the expansion of  $(3 - \frac{x^3}{6})^7$ . [3]



OR

Find the term independent of  $x$  in the expansion of  $\left(x - \frac{1}{x^2}\right)^{3n}$ .

30. Find the modulus and argument of complex number  $\frac{1}{1+i}$ . [3]

OR

Convert the complex number in the polar form:  $\sqrt{3} + i$

31. In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find the number of people who read at least one of the newspaper. [3]

#### Section D

32. A box contains 100 bulbs, 20 of which are defective. 10 bulbs are selected for inspection. Find the probability that:
- i. all 10 are defective
  - ii. all 10 are good
  - iii. at least one is defective
  - iv. none is defective

33. Differentiate  $\log \sin x$  from first principles. [5]

OR

Differentiate  $x^2 \sin x$  from first principle.

34. The Sum of two no. is 6 times their geometric mean, show that no. are in the ratio  $(3 + 3\sqrt{2}) : (3 - 2\sqrt{2})$  [5]

35. Prove that:  $\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}$ . [5]

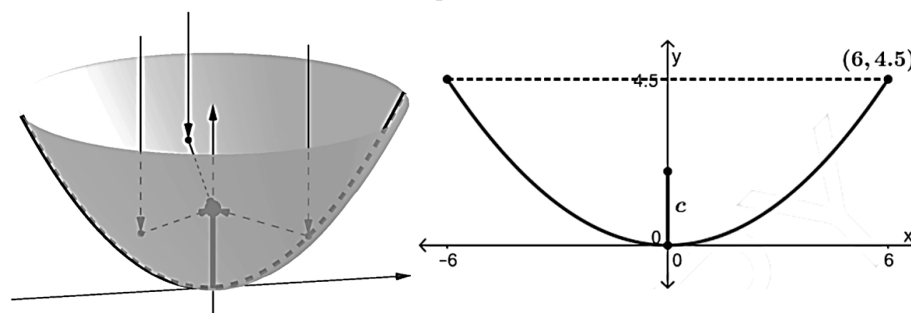
OR

Prove that:  $\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$ .

#### Section E

36. Read the following text carefully and answer the questions that follow: [4]

A satellite dish has a shape called a paraboloid, where each cross section is parabola. Since radio signals (parallel to axis) will bounce off the surface of the dish to the focus, the receiver should be placed at the focus. The dish is 12 ft across, and 4.5 ft deep at the vertex.



- i. Name the type of curve given in the above paragraph and find the equation of curve? (1)
- ii. Find the equation of parabola whose vertex is (3, 4) and focus is (5, 4). (1)
- iii. Find the equation of parabola Vertex (0, 0) passing through (2, 3) and axis is along x-axis. and also find the length of latus rectum. (2)

OR

- iv. Find focus, length of latus rectum and equation of directrix of the parabola  $x^2 = 8y$ . (2)

37. Read the following text carefully and answer the questions that follow: [4]

You are given some observations as 34, 66, 30, 38, 44, 50, 40, 60, 42, 51.

- i. Find the difference between mean deviation about the mean and mean deviation about the median. (1)
- ii. Calculate the median of the given data. (1)
- iii. The mean deviation about the mean is (2)

1. 10.0

2. 9.5

3. 9.1

4. 9.0

**OR**

Calculate the mean of the given data.(2)

38. **Read the following text carefully and answer the questions that follow:**

**[4]**

The purpose of the student council is to give students an opportunity to develop leadership by organizing and carrying out school activities and service projects. Create an environment where every student can voice out their concern or need. Raju, Ravi Joseph, Sangeeta, Priya, Meena and Aman are members of student's council. There is a photo session in a school these 7 students are to be seated in a row for photo session.



- i. Find the total number of arrangements so that Raju and Ravi are at extreme positions? (1)
- ii. Find the number of arrangements so that Joseph is sitting in the middle. (1)
- iii. Find the number of arrangements so that three girls are together. (2)

**OR**

Find the number of arrangements so that Aman and Ravi are not together? (2)



# Solution

## Section A

1.

(c)  $\frac{\sqrt{10-2\sqrt{5}}}{4}$

**Explanation:**

$$\sin^2 36^\circ = (1 - \cos^2 36^\circ) = \left\{ 1 - \frac{(\sqrt{5}+1)^2}{16} \right\} = \frac{(10-2\sqrt{5})}{16} \Rightarrow \sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

2.

(d)  $[-a, a]$

**Explanation:**

We have  $f(x) = \sqrt{a^2 - x^2}$

Clearly  $f(x)$  is defined if  $a^2 - x^2 \geq 0$

$$\Rightarrow x^2 \leq a^2$$

$$\Rightarrow -a \leq x \leq a \quad [\because a > 0]$$

$\therefore$  Domain of  $f$  is  $[-a, a]$

3.

(b)  $\frac{11}{16}$

**Explanation:**

If the numbers of nails and nuts are 6 and 10, respectively, then the numbers of rusted nails and rusted nuts are 3 and 5, respectively.

Total number of items =  $6 + 10 = 16$

Total number of rusted items =  $3 + 5 = 8$

Total number of ways of drawing one item =  ${}^{16}C_1$

Let  $R$  and  $N$  be the events where both the items drawn are rusted items and nails, respectively.

$R$  and  $N$  are not mutually exclusive events, because there are 3 rusted nails.

$$P(\text{either a rusted item or a nail}) = P(R \cup N)$$

$$= P(R) + P(N) - P(R \cap N)$$

$$= \frac{1}{84} + \frac{4}{84} = \frac{5}{84}$$

$$= \frac{6}{16} + \frac{8}{16} - \frac{3}{16} = \frac{11}{16}$$

4.

(d) 1

**Explanation:**

We know that,

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

$$\therefore \frac{|x|}{x} = \begin{cases} \frac{x}{x}, & \text{if } x \geq 0 \\ \frac{-x}{x}, & \text{if } x < 0 \end{cases} = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$$

Now, for all  $x \geq 0$  (however,  $x$  may large be).

$$\frac{|x|}{x} = 1$$

$$\therefore \lim_{x \rightarrow \infty} \frac{|x|}{x} = 1$$



5. (a)  $x + 1 = 0$ ,  $y + 1 = 0$

**Explanation:**

The lines  $x + 1 = 0$  and  $y + 1 = 0$  are perpendicular to each other.

The slope of the line  $x + y = 0$  is -1

Hence the angle made by this line with respect to X-axis is  $45^\circ$

In other words, the angle made by this line with  $x + 1 = 0$  is  $45^\circ$

Clearly the other line with which it can make  $45^\circ$  is  $y + 1 = 0$

6.

(c) A

**Explanation:**

Let us assume that  $x \in A \cap (A \cup B)$

$$\Rightarrow x \in A \text{ and } x \in (A \cup B)$$

$$\Rightarrow x \in A \text{ and } (x \in A \text{ or } x \in B)$$

$$\Rightarrow (x \in A \text{ and } x \in A) \text{ or } (x \in A \text{ and } x \in B)$$

$$\Rightarrow x \in A \text{ or } x \in A \cap B$$

$$\Rightarrow x \in A$$

$$\text{Therefore, } A \cap (A \cup B) = A$$

7.

(b)  $-3+1i$

**Explanation:**

In subtracting one complex number from other, difference of corresponding parts of two complex numbers is calculated. So,  $z_1 - z_2$  will be:

$$= (2-5) + (3-2) i$$

$$= -3+1i.$$

8.

(b) neither one-one nor onto

**Explanation:**

Given that  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function where

$$f(x) = \frac{x^2-8}{x^2+2}$$

Here, we can see that for negative as well as positive  $x$  we will get same value.

So, it is not one-one.

$$y = f(x)$$

$$\Rightarrow y = \frac{x^2-8}{x^2+2}$$

$$\Rightarrow y(x^2 + 2) = (x^2 - 8)$$

$$\Rightarrow x^2(y - 1) = -2y - 8$$

$$\Rightarrow x = \sqrt{\frac{2y+8}{1-y}}$$

For  $y = 1$ , no  $x$  is defined.

So,  $f$  is not onto.

9.

(b)  $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$

**Explanation:**

$$\text{Given } \frac{|x|-1}{|x|-2} \geq 0, x \neq \pm 2$$

$$\frac{|x|-1}{|x|-2} \geq 0$$

$$\Rightarrow |x| - 1 \geq 0 \text{ and } |x| - 2 \geq 0 \text{ or } |x| - 1 \leq 0 \text{ and } |x| - 2 \leq 0 \left[ \because \frac{a}{b} \geq 0 \Rightarrow (a \geq 0 \text{ and } b \geq 0) \text{ or } (a \leq 0 \text{ and } b \leq 0) \right]$$

$$\Rightarrow |x| \geq 1 \text{ and } |x| \geq 2 \text{ or } |x| \leq 1 \text{ and } |x| \leq 2$$

$$\Rightarrow |x| \geq 2 \text{ or } |x| \leq 1 \left[ \because |x| \geq a \Rightarrow x \geq a \text{ or } x \leq -a \text{ and } |x| \leq a \Rightarrow -a \leq x \leq a \right]$$



$$\Rightarrow x \geq 2 \text{ or } x \leq -2 \text{ or } -1 \leq x \leq 1$$

$$\Rightarrow x \in (2, \infty) \text{ or } x \in (-\infty, -2) \text{ or } x \in (-1, 1)$$

$$\Rightarrow x \in (2, \infty) \cup (-\infty, -2) \cup (-1, 1)$$

10.

(b)  $\frac{(\sqrt{3}+1)}{2}$

**Explanation:**

Using  $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$ , we get:

$$2 \cos 45^\circ \cos 15^\circ = \cos(45^\circ + 15^\circ) + \cos(45^\circ - 15^\circ)$$

$$= (\cos 60^\circ + \cos 30^\circ) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \frac{(\sqrt{3}+1)}{2}$$

11.

(d)  $A \cap B^c$

**Explanation:**

$$A \cap B^c$$

A and B are two sets.

$A \cap B$  is the common region in both the sets.

$(A \cap B^c)$  is all the region in the universal set except  $A \cap B$

$$\text{Now, } A \cap (A \cap B)^c = A \cap B^c$$

12.

(b) 12th

**Explanation:**

Given GP is  $\sqrt{3}, 3, 3\sqrt{3} \dots$

$$\text{Here, we have } a = \sqrt{3} \text{ and } r = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

$$\text{Let } T_n = 729. \text{ Then, } ar^{n-1} = 729 \Rightarrow \sqrt{3} \times (\sqrt{3})^{n-1} = 729 = 3^6$$

$$\therefore (\sqrt{3})^n = (\sqrt{3})^{12} \Rightarrow n = 12.$$

13.

(b)  $330m^4$

**Explanation:**

We have the general term of  $(x+a)^n$  is  $T_{r+1} = {}^nC_r (x)^{n-r} a^r$

$$\text{Now consider } \left(x - \frac{m}{x}\right)^{11}$$

$$\text{Here } T_{r+1} = {}^{11}C_r (x)^{11-r} \left(-\frac{m}{x}\right)^r$$

Comparing the indices of x in  $x^3$  and in  $T_{r+1}$ , we get

$$11 - r - r = 3 \Rightarrow r = 4$$

$$\text{Therefore the required term is } T_{4+1} = T_5 = {}^{11}C_4 (x)^{11-4} \left(-\frac{m}{x}\right)^4 = 330m^4 x^3$$

14. (a)  $(-\infty, -7) \cup (7, \infty)$

**Explanation:**

$$|x| > 7$$

$$\Rightarrow -7 > x > 7$$

$$\Rightarrow x < -7 \text{ or } x > 7 \text{ [} \because |x| > a \Leftrightarrow x < -a \text{ or } x > a \text{]}$$

$$\Rightarrow x \in (-\infty, -7) \text{ or } x \in (7, \infty)$$

$$\Rightarrow x \in (-\infty, -7) \cup (7, \infty)$$

15.

(b)  $A \subseteq B$

**Explanation:**



$$A \subseteq B$$

$\Rightarrow$  since set A is totally contained in Set B.

16.

**(b)**  $-\sqrt{2}$

**Explanation:**

Since x lies in quadrant III, we have:  $\pi < x < \frac{3\pi}{2}$

$$\therefore \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \frac{x}{2} \text{ lies in quadrant II}$$

$$\Rightarrow \sin \frac{x}{2} > 0, \cos \frac{x}{2} < 0 \text{ and } \tan \frac{x}{2} < 0$$

$$2 \sin^2 \frac{x}{2} = (1 - \cos x) = \left(1 + \frac{1}{3}\right) = \frac{4}{3} \Rightarrow \sin^2 \frac{x}{2} = \frac{2}{3}$$

$$\therefore \sin \frac{x}{2} = \sqrt{\frac{2}{3}}$$

$$2 \cos^2 \frac{x}{2} = (1 + \cos x) = \left(1 - \frac{1}{3}\right) = \frac{2}{3} \Rightarrow \cos^2 \frac{x}{2} = \frac{1}{3}$$

$$\therefore \cos \frac{x}{2} = -\frac{1}{\sqrt{3}}$$

$$\therefore \tan \frac{x}{2} = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{\sqrt{2}}{\sqrt{3}} \times (-\sqrt{3}) = -\sqrt{2}$$

17.

**(b)**  $-\bar{z}_2$

**Explanation:**

Let  $z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$  and  $z_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$ , where  $r_1 = |z_1|$ ,  $r_2 = |z_2|$  and  $\theta_1, \theta_2$  are the amplitude of the complex numbers.

$$\text{Given } |z_1| = |z_2| \text{ and } \text{Amp. } z_1 + \text{Amp. } z_2 = \pi \Rightarrow r_1 = r_2 \text{ and } \theta_1 + \theta_2 = \pi$$

$$\text{Now } z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$$

$$\Rightarrow z_1 = r_2 [\cos(\pi - \theta_2) + i \sin(\pi - \theta_2)]$$

$$\Rightarrow z_1 = r_2 (-\cos \theta_2 + i \sin \theta_2) = -[r_2 (\cos \theta_2 - i \sin \theta_2)] = -\bar{z}_2$$

18. **(a)** 63

**Explanation:**

There are three multiple choice question, each has four possible answers. Thus, the total number of possible answers will be  $4 \times 4 \times 4 = 64$ . Out of these possible answer only one will be correct and therefore, the number of ways in which a student can fail to get correct answer is  $64 - 1 = 63$ .

19.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:**

Assertion is true

Reason!

$$(1 - 3x + 3x^2 - x^3)^{20}$$

$$= [(1 - x)^3]^{20}$$

$$= (1 - x)^{60}$$

$$\therefore \text{No. of dissimilar terms in the expansion of } (1 - 3x + 3x^2 - x^3)^{20} \text{ is } 61$$

20.

**(c)** A is true but R is false.

**Explanation:**

**Assertion** Mean of the given series

$$\bar{x} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n}$$

$$= \frac{4+7+8+9+10+12+13+17}{8} = 10$$

| $x_i$           | $ x_i - \bar{x} $           |
|-----------------|-----------------------------|
| 4               | $ 4 - 10  = 6$              |
| 7               | $ 7 - 10  = 3$              |
| 8               | $ 8 - 10  = 2$              |
| 9               | $ 9 - 10  = 1$              |
| 10              | $ 10 - 10  = 0$             |
| 12              | $ 12 - 10  = 2$             |
| 13              | $ 13 - 10  = 3$             |
| 17              | $ 17 - 10  = 7$             |
| $\sum x_i = 80$ | $\sum  x_i - \bar{x}  = 24$ |

$\therefore$  Mean deviation about mean

$$= \frac{\sum |x_i - \bar{x}|}{n} = \frac{24}{8} = 3$$

**Reason** Mean of the given series

$$\begin{aligned}\bar{x} &= \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{\sum x_i}{n} \\ &= \frac{38+70+48+40+42+55}{+63+46+54+44} = 50\end{aligned}$$

$\therefore$  Mean deviation about mean

$$\begin{aligned}&= \frac{\sum |x_i - \bar{x}|}{n} \\ &= \frac{84}{10} = 8.4\end{aligned}$$

Hence, Assertion is true and Reason is false.

### Section B

21. Here, some of the first set element has same image in second set.

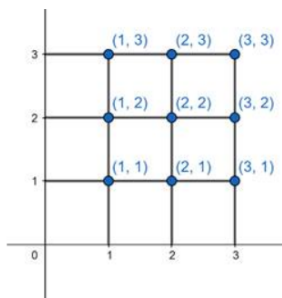
$\therefore$  g is not a function

OR

According to the question,

To find:  $B \times B$

$$B \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$



22. Let  $f(x) = 3x^2 + 2x - 5$

We need to find the derivative of  $f(x)$  i.e.  $f'(x)$

We know that by using first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = 3x^2 + 2x - 5$$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 5$$

$$= 3(x^2 + h^2 + 2xh) + 2x + 2h - 5$$

$$[\because (a+b)^2 = a^2 + b^2 + 2ab]$$

$$= 3x^2 + 3h^2 + 6xh + 2x + 2h - 5$$

Put values in (i), we get

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - (3x^2 + 2x - 5)}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{3x^2 + 3h^2 + 6xh + 2x + 2h - 5 - 3x^2 - 2x + 5}{h} \\
&= \lim_{h \rightarrow 0} \frac{3h^2 + 6xh + 2h}{h} \\
&= \lim_{h \rightarrow 0} 3h + 6x + 2 \\
f'(x) &= 3(0) + 6x + 2 \\
&= 6x + 2 \\
\text{Hence, } f'(x) &= 6x + 2
\end{aligned}$$

23. The given equation of circle is

$$2x^2 + 2y^2 - x = 0 \Rightarrow x^2 + y^2 - \frac{x}{2} = 0$$

$$\Rightarrow \left(x^2 - \frac{x}{2}\right) + y^2 = 0$$

On adding  $\frac{1}{16}$  to make perfect squares, we get

$$\left(x^2 - \frac{x}{2} + \frac{1}{16}\right) + y^2 = \frac{1}{16}$$

$$\Rightarrow \left(x - \frac{1}{4}\right)^2 + (y - 0)^2 = \left(\frac{1}{4}\right)^2$$

On comparing with  $(x - h)^2 + (y - k)^2 = r^2$ , we get

$$h = \frac{1}{4}, k = 0 \text{ and } r = \frac{1}{4}$$

$$\therefore \text{Centre} = (h, k) = \left(\frac{1}{4}, 0\right)$$

$$\text{and Radius} = \frac{1}{4}$$

OR

Here foci are  $(0, \pm 13)$  which lie on y-axis.

So the equation of hyperbola in standard form is  $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$$\therefore (13)^2 = a^2 + (12)^2$$

$$\Rightarrow a^2 = 169 - 144 = 25$$

Thus required equation of hyperbola is

$$\frac{y^2}{25} - \frac{x^2}{(12)^2} = 1 \Rightarrow \frac{y^2}{25} - \frac{x^2}{144} = 1$$

24. Here,  $B - C$  = represents all elements in B that are not in C

$$B \cap C = \{e, g\}$$

$$A - (B \cap C) = \{a, b, c, d\} \dots (1)$$

$$(A - B) = \{b, d\}$$

$$(A - C) = \{a, c, d\}$$

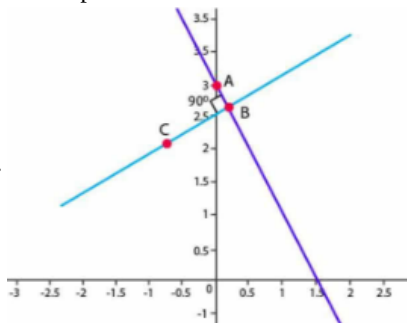
$$(A - B) \cap (A - C) = \{a, b, c, d\} \dots (2),$$

From (1) and (2)

$$\Rightarrow A - (B \cap C) = (A - B) \cap (A - C)$$

Hence proved.

25.



Here, it is given: The given line is  $x - 2y + 5 = 0$ . The line perpendicular to this given line passes through  $(0, 3)$

Using formula: The product of slopes of two perpendicular lines =  $-1$ .

The slope of this line is  $1/2$ .

$$\therefore \text{the slope of the perpendicular line} = \frac{-1}{1/2} = -2$$

The equation of the line can be written in the form  $y = (-2)x + c$

(c is the y-intercept)

This line passes through  $(0, 3)$  so the point will satisfy the equation of the line.

$$\therefore 3 = (-2)0 + c \text{ i.e. } c = 3$$

Therefore, the required equation is  $y = -2x + 3$   
 i.e.  $2x + y = 3$

### Section C

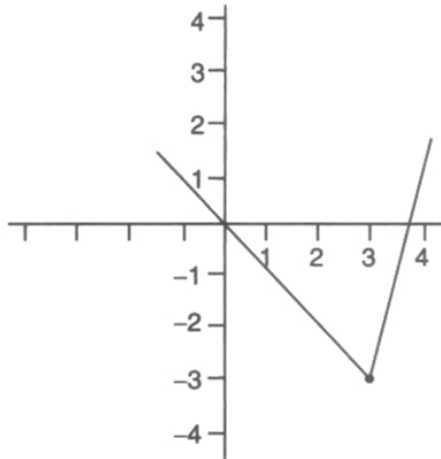
26. Given,  $f(x) = |2x - 3| - 3$

The domain of the expression is all real number except where the expression is undefined. In this case, there is not real number that makes the expression undefined.

$\therefore$  Domain of  $f = (-\infty, \infty) = \mathbb{R}$

The absolute value of expression has a 'V' shape. The range of a positive absolute value expression starts at its vertex and extends to infinity.

Range of  $f = [-3, \infty)$  or  $\{y : y \geq -3\}$



27. Here  $3x - 7 > 5x - 1$

$$\Rightarrow 3x - 5x > -1 + 7$$

$$\Rightarrow -2x > 6$$

Dividing both sides by -2, we have

$$\frac{-2x}{-2} < \frac{6}{-2} \Rightarrow x < -3$$

Thus the solution set is  $(-\infty, -3)$ .

28. Let  $A(0, 7, -10)$ ,  $B(1, 6, -6)$  and  $C(4, 9, -6)$  be three vertices of triangle ABC. Then

$$AB = \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$$

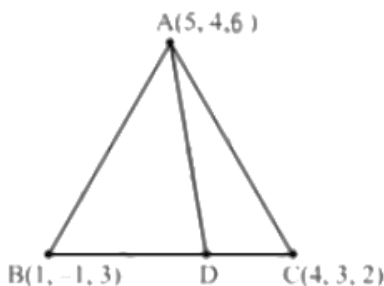
$$BC = \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} = \sqrt{9+9+0} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(4-0)^2 + (9-7)^2 + (-6+10)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$$

Now  $AB = BC$

Thus, ABC is an isosceles triangle.

OR



$$AB = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{16+25+9} = \sqrt{50} = 5\sqrt{2}$$

$$BC = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{18} = 3\sqrt{2}$$

AD is the internal bisector of  $\angle BAC$

$$\therefore \frac{BD}{DC} = \frac{AB}{AC} = \frac{5}{3}$$

Thus, point D divides BC internally in the ratio 5 : 3

$$\therefore D = \left( \frac{5 \times 4 + 3 \times 1}{5+3}, \frac{5 \times 3 + 3 \times (-1)}{5+3}, \frac{5 \times 2 + 3 \times 3}{5+3} \right)$$

$$\Rightarrow D = \left( \frac{23}{8}, \frac{12}{8}, \frac{19}{8} \right)$$

$$\Rightarrow D = \left( \frac{23}{8}, \frac{3}{2}, \frac{19}{8} \right)$$

$$\therefore AD = \sqrt{\left( 5 - \frac{23}{8} \right)^2 + \left( 4 - \frac{12}{8} \right)^2 + \left( 6 - \frac{19}{8} \right)^2}$$

$$= \sqrt{\frac{17^2 + 20^2 + 29^2}{8^2}}$$

$$= \sqrt{\frac{289 + 400 + 841}{8^2}}$$

$$AD = \frac{\sqrt{1530}}{8}$$

29. Here,  $n = 7$  (odd)

$\therefore$  Middle terms are obtained  $\left( \frac{n+1}{2} \right)$ th and  $\left( \frac{n+3}{2} \right)$ th terms i.e.,  $\left( \frac{7+1}{2} \right)$ th term and  $\left( \frac{7+3}{2} \right)$ th term i.e., 4th and 5th terms.

$$\text{Now, } T_4 = T_{3+1} = {}^7C_3 (3)^{7-3} \left( -\frac{x^3}{6} \right)^3$$

$$[\because T_{r+1} = {}^nC_r x^{n-r} y^r]$$

$$= \frac{7 \times 6 \times 5}{6} \times \frac{3^4 (-1)^3 \times x^9}{6^3}$$

$$= \frac{35 \times 81 \times (-1) \times x^9}{6 \times 6 \times 6}$$

$$= -\frac{35 \times 27 \times x^9}{2 \times 6 \times 6} = -\frac{105}{8} x^9$$

$$\text{and } T_5 = T_{4+1} = {}^7C_4 (3)^{7-4} \left( -\frac{x^3}{6} \right)^4$$

$$= {}^7C_3 \frac{3^3 (-1)^4 (x^3)^4}{6^4} [\because {}^nC_r = {}^nC_{r-1}]$$

$$= \frac{7 \times 6 \times 5}{6} \times \frac{3 \times 3 \times 3 \times x^{12}}{6 \times 6 \times 6 \times 6} = \frac{35}{48} x^{12}$$

OR

To find: term independent of  $x$ , i.e.  $x^0$

$$\text{For } \left( x - \frac{1}{x^2} \right)^{3n}$$

we have,  $a = x$ ,  $b = -\frac{1}{x^2}$  and  $N = 3n$

Therefore, We have a formula,

$$t_{r+1} = \left( \frac{N}{r} \right) a^{N-r} b^r$$

$$= \left( \frac{3n}{r} \right) (x)^{3n-r} \left( -\frac{1}{x^2} \right)^r$$

$$= \left( \frac{3n}{r} \right) (x)^{3n-r} (-1)^r \left( \frac{1}{x^2} \right)^r$$

$$= \left( \frac{3n}{r} \right) (x)^{3n-r} (-1)^r (x)^{-2r}$$

$$= \left( \frac{3n}{r} \right) (-1)^r (x)^{3n-r-2r}$$

$$= \left( \frac{3n}{r} \right) (-1)^r (x)^{3n-3r}$$

Now, to get coefficient of term independent of  $x$ , that is coefficient of  $x_0$  we must have,

$$(x)^{3n-3r} = x^0$$

$$3n - 3r = 0$$

$$3r = 3n$$

$$r = n$$

Therefore, coefficient of  $x^0 = \left( \frac{3n}{n} \right) (-1)^n$ .

$$30. \text{ We have, } \frac{1}{1+i} = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{1^2-i^2} = \frac{1-i}{1+1}$$

$$= \frac{1-i}{2} = \frac{1}{2} - \frac{i}{2}$$

$$\text{Let } r \cos \theta = \frac{1}{2} \dots (i)$$

$$\text{and } r \sin \theta = -\frac{1}{2} \dots (ii)$$

On squaring and adding Eqs. (i) and (ii), we get

$$r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left( \frac{1}{2} \right)^2 + \left( -\frac{1}{2} \right)^2$$

$$\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \Rightarrow r = \frac{1}{\sqrt{2}} [\because r \text{ is positive}]$$

On putting the value of  $r$  in Eqs. (i) and (ii), we get

$$\cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}$$

Since,  $\cos \theta$  is positive and  $\sin \theta$  is negative.

So,  $\theta$  lies in IV quadrant.

$$\therefore \theta = -\frac{\pi}{4}$$

Hence, modulus of  $\frac{1}{1+i}$  is  $\frac{1}{\sqrt{2}}$  and argument is  $\frac{-\pi}{4}$ .

OR

$$\text{Here } z = \sqrt{3} + i = r(\cos \theta + i \sin \theta)$$

$$\Rightarrow r \cos \theta = \sqrt{3} \text{ and } r \sin \theta = 1$$

Squaring both sides of (i) and adding

$$r^2(\cos^2 \theta + \sin^2 \theta) = 3 + 1 \Rightarrow r^2 = 4 \Rightarrow r = 2$$

$$\therefore 2 \cos \theta = \sqrt{3} \text{ and } 2 \sin \theta = 1$$

$$\therefore \cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}$$

Since  $\sin \theta$  and  $\cos \theta$  are both positive

$\therefore \theta$  lies in first quadrant

$$\therefore \theta = \frac{\pi}{6}$$

Hence polar form of  $z$  is  $2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$

31. Here

$$n(U) = a + b + c + d + e + f + g + h = 60 \dots(i)$$

$$n(H) = a + b + c + d = 25 \dots(ii)$$

$$n(T) = b + c + f + g = 26 \dots(iii)$$

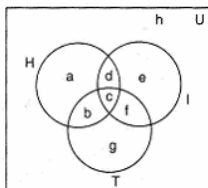
$$n(I) = c + d + e + f = 26 \dots(iv)$$

$$n(H \cap I) = c + d = 9 \dots(v)$$

$$n(H \cap T) = b + c = 11 \dots(vi)$$

$$n(T \cap I) = c + f = 8 \dots(vii)$$

$$n(H \cap T \cap I) = c = 3 \dots(viii)$$



Putting value of  $c$  in (vii),

$$3 + f = 8 \Rightarrow f = 5$$

Putting value of  $c$  in (vi),

$$3 + b = 11 \Rightarrow b = 8$$

Putting values of  $c$  in (v),

$$3 + d = 9 \Rightarrow d = 6$$

Putting value of  $c, d, f$  in (iv),

$$3 + 6 + e + 5 = 26 \Rightarrow e = 26 - 14 = 12$$

Putting value of  $b, c, f$  in (iii),

$$8 + 3 + 5 + g = 26 \Rightarrow g = 26 - 16 = 10$$

Putting value of  $b, c, d$  in (ii),

$$a + 8 + 3 + 6 = 25 \Rightarrow a = 25 - 17 = 8$$

Number of people who read at least one of the three newspapers

$$= a + b + c + d + e + f + g$$

$$= 8 + 8 + 3 + 6 + 12 + 5 + 10 = 52$$

## Section D

32. Given,

A box containing 100 bulbs, out of which 20 are defective

$$\therefore \text{Number of good bulbs} = 100 - 20 = 80$$

10 bulbs are selected for inspection

$\therefore$  Numbers of elementary events in sample space,

$$n(s) = {}^{100}C_{10}$$

i. Suppose  $E$  be the event that all 10 bulbs selected are defective.

$$n(E) = {}^{20}C_{10}$$

$$\therefore P(E) = \frac{{}^{20}C_{10}}{{}^{100}C_{10}}$$

ii. Suppose E be the event that all 10 good bulbs are selected.

$$\therefore n(E) = {}^{80}C_{10}$$

$$\therefore P(E) = \frac{{}^{80}C_{10}}{{}^{100}C_{10}}$$

iii. Suppose E be the event that at least one bulb is defective.

Let e be the event that none of the bulbs are defective.

$$\therefore n(e) = {}^{80}C_{10}$$

$$\therefore p(e) = \frac{{}^{80}C_{10}}{{}^{100}C_{10}}$$

$$\therefore P(E) = 1 - p(e)$$

$$= 1 - \frac{{}^{80}C_{10}}{{}^{100}C_{10}}$$

iv. Suppose E be the event that none of the selected bulbs is defective, that is all bulbs are good, so,

$$n(E) = {}^{80}C_{10}$$

$$P(E) = \frac{{}^{80}C_{10}}{{}^{100}C_{10}}$$

33. Let  $f(x) = \log \sin x$ . Then,  $f(x+h) = \log \sin(x+h)$

$$\begin{aligned} \therefore \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \sin(x+h) - \log \sin x}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ \frac{\sin(x+h)}{\sin x} \right\}}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\sin(x+h) - \sin x}{\sin x} \right\}}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\sin(x+h) - \sin x}{\sin x} \right\}}{h} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\sin(x+h) - \sin x}{\sin x} \right\}}{h \left\{ \frac{\sin(x+h) - \sin x}{\sin x} \right\}} \times \left\{ \frac{\sin(x+h) - \sin x}{\sin x} \right\} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\sin(x+h) - \sin x}{\sin x} \right\}}{\left\{ \frac{\sin(x+h) - \sin x}{\sin x} \right\}} \times \frac{\sin(x+h) - \sin x}{h} \times \frac{1}{\sin x} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\sin(x+h) - \sin x}{\sin x} \right\}}{\left\{ \frac{\sin(x+h) - \sin x}{\sin x} \right\}} \times \lim_{h \rightarrow 0} \frac{2 \sin \frac{h}{2} \cos \left( x + \frac{h}{2} \right)}{h} \times \frac{1}{\sin x} \\ \Rightarrow \frac{d}{dx}(f(x)) &= \lim_{h \rightarrow 0} \frac{\log \left\{ 1 + \frac{\sin(x+h) - \sin x}{\sin x} \right\}}{\left\{ \frac{\sin(x+h) - \sin x}{\sin x} \right\}} \times \lim_{h \rightarrow 0} \frac{\sin \left( \frac{h}{2} \right) \cos \left( x + \frac{h}{2} \right)}{\frac{h}{2}} \times \frac{1}{\sin x} \\ \Rightarrow \frac{d}{dx}(f(x)) &= 1 \times \cos x \times \frac{1}{\sin x} = \cot x. \end{aligned}$$

OR

We have to find derivative of  $f(x) = x^2 \sin x$

Derivative of a function  $f(x)$  is given by  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  {where  $h$  is a very small positive number}

$\therefore$  Derivative of  $f(x) = x^2 \sin x$  is given as  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$\Rightarrow f(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$$

Using  $(a+b)^2 = a^2 + 2ab + b^2$ , we get

$$\Rightarrow f(x) = \lim_{h \rightarrow 0} \frac{h^2 \sin(x+h) + x^2 \sin(x+h) + 2hx \sin(x+h) - x^2 \sin x}{h}$$

Using the algebra of limits, we have

$$\begin{aligned} \Rightarrow f(x) &= \lim_{h \rightarrow 0} \frac{h^2 \sin(x+h)}{h} + \lim_{h \rightarrow 0} \frac{x^2 \sin(x+h) - x^2 \sin x}{h} + \lim_{h \rightarrow 0} \frac{2hx \sin(x+h)}{h} \\ \Rightarrow f(x) &= \lim_{h \rightarrow 0} h \sin(x+h) + \lim_{h \rightarrow 0} \frac{x^2 (\sin(x+h) - \sin x)}{h} + \lim_{h \rightarrow 0} 2x \sin(x+h) \end{aligned}$$

$$\Rightarrow f'(x) = 0 \times \sin(x+0) + 2x \sin(x+0) + x^2 \lim_{h \rightarrow 0} \frac{(\sin(x+h) - \sin x)}{h}$$

$$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{(\sin(x+h) - \sin x)}{h}$$

Using the algebra of limits we have

$$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{(\sin(x+h) - \sin x)}{h}$$

We can't evaluate the limits at this stage only as on putting value it will take  $\frac{0}{0}$  form. So, we need to do little modifications.

$$\text{Use: } \sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$$

$$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{h}{2}\right)}{h}$$

$$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{\cos\left(x + \frac{h}{2}\right) \sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

Using the algebra of limits:

$$\Rightarrow f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \times \lim_{h \rightarrow 0} \cos\left(x + \frac{h}{2}\right)$$

$$\text{By using the formula we get } - \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\therefore f'(x) = 2x \sin x + x^2 \lim_{h \rightarrow 0} \cos\left(x + \frac{h}{2}\right)$$

substitute the value of h to evaluate the limit:

$$\text{Therefore, } f'(x) = 2x \sin x + x^2 \cos(x+0) = 2x \sin x + x^2 \cos x$$

Hence,

$$\text{Derivative of } f(x) = (x^2 \sin x) \text{ is } (2x \sin x + x^2 \cos x)$$

$$34. a + b = 6\sqrt{ab}$$

$$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1}$$

by C and D

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3+1}{3-1}$$

$$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{2}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}}{1}$$

again by C and D

$$\frac{\sqrt{a}+\sqrt{b}+\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}-\sqrt{a}-\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$$\frac{a}{b} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2} \text{ (on squaring both sides)}$$

$$\frac{a}{b} = \frac{2+1+2\sqrt{2}}{2+1-2\sqrt{2}}$$

$$\frac{a}{b} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$a : b = (3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$$

$$35. \text{ We have to prove that } \sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ = \frac{1}{16}.$$

$$\text{LHS} = \sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ$$

By regrouping the LHS and multiplying and dividing by 4 we get,

$$= \frac{1}{4} (2 \sin 66^\circ \sin 6^\circ) (2 \sin 78^\circ \sin 42^\circ)$$

$$\text{But } 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Then the above equation becomes,

$$= \frac{1}{4} (\cos(66^\circ - 6^\circ) - \cos(66^\circ + 6^\circ)) (\cos(78^\circ - 42^\circ) - \cos(78^\circ + 42^\circ))$$

$$= \frac{1}{4} (\cos(60^\circ) - \cos(72^\circ)) (\cos(36^\circ) - \cos(120^\circ))$$

$$= \frac{1}{4} (\cos(60^\circ) - \cos(90^\circ - 18^\circ)) (\cos(36^\circ) - \cos(180^\circ - 120^\circ))$$

$$\text{But } \cos(90^\circ - \theta) = \sin \theta \text{ and } \cos(180^\circ - \theta) = -\cos(\theta).$$

Then the above equation becomes,



$$= \frac{1}{4} (\cos(60^\circ) - \cos(18^\circ)) (\cos(36^\circ) + \cos(60^\circ))$$

$$\text{Now, } \cos(36^\circ) = \frac{\sqrt{5}+1}{4}$$

$$\sin(18^\circ) = \frac{\sqrt{5}-1}{4}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Substituting the corresponding values, we get

$$= \frac{1}{4} \left( \frac{1}{2} - \frac{\sqrt{5}-1}{4} \right) \left( \frac{\sqrt{5}+1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left( \frac{2-\sqrt{5}+1}{4} \right) \left( \frac{\sqrt{5}+1}{4} + \frac{1}{2} \right)$$

$$= \frac{1}{4} \left( \frac{3-\sqrt{5}}{4} \right) \left( \frac{3+\sqrt{5}}{4} \right)$$

$$= \frac{1}{4} \left( \frac{3^2 - (\sqrt{5})^2}{4 \times 4} \right)$$

$$= \frac{1}{4} \left( \frac{9-5}{16} \right)$$

$$= \frac{1}{16}$$

LHS = RHS

Hence proved.

OR

$$\cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ = \frac{3}{16}$$

$$\text{LHS} = \cos 10^\circ \cos 30^\circ \cos 50^\circ \cos 70^\circ$$

$$= \cos 30^\circ \cos 10^\circ \cos 50^\circ \cos 70^\circ$$

$$= \frac{\sqrt{3}}{2} (\cos 10^\circ \cos 50^\circ \cos 70^\circ)$$

$$= \frac{\sqrt{3}}{2} (\cos 10^\circ \cos 50^\circ) \cos 70^\circ$$

$$= \frac{\sqrt{3}}{4} (2 \cos 10^\circ \cos 50^\circ) \cos 70^\circ \text{ [Multiplying and dividing by 2]}$$

$$= \frac{\sqrt{3}}{4} \cos 70^\circ \{ \cos (50^\circ + 10^\circ) + \cos (10^\circ - 50^\circ) \} \text{ [Using } 2 \cos A \cos B = \cos (A + B) + \cos (A - B)]$$

$$= \frac{\sqrt{3}}{4} \cos 70^\circ \{ \cos 60^\circ + \cos (-40^\circ) \}$$

$$= \frac{\sqrt{3}}{4} \cos 70^\circ \left[ \frac{1}{2} + \cos 40^\circ \right] \text{ [}\because \cos 60^\circ = \frac{1}{2} \text{ and } \cos (-x) = \cos x]$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{4} \cos 70^\circ \cos 40^\circ$$

$$= \frac{\sqrt{3}}{8} \cos 70^\circ + \frac{\sqrt{3}}{8} (2 \cos 70^\circ \cos 40^\circ)$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos (70^\circ + 40^\circ) + \cos (70^\circ - 40^\circ)]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos 110^\circ + \cos 30^\circ]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ + \cos (180^\circ - 70^\circ) + \frac{\sqrt{3}}{2}] \text{ [}\because \cos 30^\circ = \frac{\sqrt{3}}{2}]$$

$$= \frac{\sqrt{3}}{8} [\cos 70^\circ - \cos 70^\circ + \frac{\sqrt{3}}{2}] \text{ [}\because \cos (180^\circ - x) = -\cos x]$$

$$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$$

= RHS

Hence proved.

### Section E

36. i. Given curve is a parabola

$$\text{Equation of parabola is } x^2 = 4ay$$

It passes through the point (6, 4.5)

$$\Rightarrow 36 = 4 \times a \times 4.5$$

$$\Rightarrow 36 = 18a$$

$$\Rightarrow a = 2$$

$$\text{Equation of parabola is } x^2 = 8y$$

ii. Distance between focus and vertex is  $a = \sqrt{(4-4)^2 + (5-3)^2} = 2$

$$\text{Equation of parabola is } (y-k)^2 = 4a(x-h)$$

where (h, k) is vertex

$$\Rightarrow \text{Equation of parabola with vertex (3, 4) \& } a = 2$$

$$\Rightarrow (y-4)^2 = 8(x-3)$$

iii. Equation of parabola with axis along x - axis

$$y^2 = 4ax$$

which passes through (2, 3)

$$\Rightarrow 9 = 4a \times 2$$

$$\Rightarrow 4a = \frac{9}{2}$$

hence required equation of parabola is

$$y^2 = \frac{9}{2}x$$

$$\Rightarrow 2y^2 = 9x$$

Hence length of latus rectum =  $4a = 4.5$

**OR**

$$x^2 = 8y$$

$$a = 2$$

Focus of parabola is (0, 2)

length of latus rectum is  $4a = 4 \times 2 = 8$

Equation of directrix  $y + 2 = 0$

37. i. = mean deviation about the mean - mean deviation about the median

$$= 9.0 - 8.7$$

$$= 0.3$$

ii. Number of observation are given calculate mean deviation Mean deviation  $\sum \frac{d_i}{n}$

Here Observation 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Since Median is the middle number of all the observation

Arrange the numbers in Ascending orders we get 30, 34, 38, 40, 44, 50, 51, 60, 66

Here the number of observation are Even then the middle terms are 42, 44...

Therefore, the median =  $\frac{42+44}{2}$

$$= \frac{86}{2} = 43$$

iii. Mean deviation is

| $x_i$ | $ d_i  =  x_i - 43 $ |
|-------|----------------------|
| 30    | 13                   |
| 34    | 9                    |
| 38    | 5                    |
| 40    | 3                    |
| 42    | 1                    |
| 44    | 1                    |
| 50    | 7                    |
| 51    | 8                    |
| 60    | 17                   |
| 66    | 23                   |
| Total | 87                   |

$$MD = \frac{1}{n} \sum_{i=1}^n |d_i|$$

$$= \frac{90}{10}$$

$$= 9$$

**OR**

The observation are 34, 66, 30, 38, 44, 50, 40, 60, 42, 51

$$\text{Mean} = \frac{\sum x_i}{n}$$

$$= \frac{34+66+30+38+44+50+40+60+42+51}{10}$$

$$= \frac{455}{10}$$

$$= 45.5$$

38. i. Given Raju and Ravi are at the extreme positions

**Case 1** Raju \_\_\_\_\_ Ravi

**Case 2** Ravi \_\_\_\_\_ Raju

So remaining 5 places are filled in 5! Ways in both cases

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Hence total number of arrangements =  $120 \times 2 = 240$  ways

ii. \_\_\_\_\_ **Joseph** \_\_\_\_\_

So here middle place is occupied by Joseph remaining 6 places are filled by remaining 6 students in 6! Ways

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ ways}$$

iii. When all girls are together let's consider them as a single unit. So four 4 boys with single group of girls can be arranged in  $4 + 1 = 5!$  Ways

\_\_\_\_\_ Sangeeta Priya Meena

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

But all the three girls can be arranged in themselves in 3! Ways =  $3 \times 2 \times 1 = 6$

Hence total number of ways =  $5! \times 3! = 120 \times 6 = 720$

**OR**

When Aman and Ravi are together let's consider them as a single unit. So remaining 5 students with single group of Aman and Ravi can be arranged in  $5 + 1 = 6!$  Ways

\_\_\_\_\_ Aman Ravi

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$$

But Aman and Ravi can be arranged in themselves in 2! Ways =  $2 \times 1 = 2$

Hence total number of ways =  $6! \times 2! = 720 \times 2 = 1440$  ways ...(i)

Total number of sitting arrangements of all 7 students without restriction

\_\_\_\_\_

All seven students can fill seven seats in 7! Ways

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040 \text{ ways} \dots(ii)$$

But here we need the number of arrangements so that Aman and Ravi are not together = Total number of sitting arrangements of all 7 students without restriction - Number of arrangements so that Aman and Ravi are together.

From (i) and (ii) we have

The number of arrangements so that Aman and Ravi are not together =  $5040 - 1440 = 3600$